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CALCULATION OF HEAT TRANSFER IN TURBULENT FLOW WITH ALLOWANCE FOR SECONDARY FLOW

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Аннотация—Излагается метод расчёта поля температуры в турбулентном потоке жидкости в каналах некруглого поперечного сечения. Приводятся результаты расчёта для случая турбулентного течения жидкости в квадратном канале. Для сравнения приведены результаты измерений поля температуры в потоке жидкости в усло — виях, близких к расчётным.

NOMENCLATURE

- $2b$, side of square duct [m];
- x, y, coordinates in plane normal to axis of duct [m];
- Z, coordinates in direction of duct axis [m];
- ξ_1, $= (x/b)$, $\xi_2 = (y/b)$, dimensionless coordinates in plane normal to axis of duct;
- u, v, flow velocity components in plane normal to axis of duct x, y [m/s];
- w, longitudinal velocity component [m/s];
- w_0, longitudinal velocity component at the point $\xi_1 = \xi_2 = 0$ [m/s];
- U, $= (u/w_0)$, $V = (v/w_0)$, dimensionless velocity components in plane normal to axis of duct;
- W, $= (w/w_0)$, dimensionless longitudinal velocity component;
- t, scalar property, transferred by turbulent flow;
- T, $= (t\lambda/qb)$, dimensionless temperature;
- a, molecular thermal diffusivity of fluid [m²/s];
- $\varepsilon_x, \varepsilon_y$, turbulent components of thermal diffusivity along the axes x and y, respectively [m²/s];
- λ, thermal conductivity of fluid [W/m deg];
- ν, kinematic viscosity of fluid [m²/s];
- q, heat transfer through duct side [W/m²];
- Re, $= (w_0 b/\nu)$, Reynolds number;
- Pe, $= (w_0 b/a)$, Péclét number;
- h, grid pitch;
- σ, relaxation parameter.

1. INTRODUCTION

THE PROBLEM of transfer of scalar property in turbulent flow is one of the more complex problems in the theory of turbulence. The cases of homogeneous isotropic turbulence [1] and of flow in channels of axial symmetry [2] are those being most frequently investigated. The turbulent flow in noncircular ducts has a peculiarity that lies in existence of the so-called "secondary flow" in a plane of the duct cross section, the intensity of which depends on the shape and size of the duct and on the flow regime.

References [3–5] are concerned with experimental investigation of the secondary flow. The calculation of hydrodynamic characteristics of the secondary flow is given in [6]. A solution of the problem of transfer of scalar property in fluid flow with inclusion of secondary flow means that the corresponding thermophysical calculations may be performed with a higher degree of accuracy and reliability.

2. THE PROBLEM STATEMENT

The problem has been solved of calculation of a temperature field in a turbulent flow within a square duct. For this case there exist some results of measurements of the temperature field in water and mercury flows [7], which make it possible to compare the experimental results with predicted data.

Herein is considered the case of a steady-state flow, when the transfer of the scalar property t in a turbulent flow is described by the equation

$$U\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} + w\frac{\partial t}{\partial z} - \frac{\partial}{\partial x}(a + \varepsilon_x)\frac{\partial t}{\partial x} - \frac{\partial}{\partial y}(a + \varepsilon_y)\frac{\partial t}{\partial y} = 0. \quad (1)$$

The terms $u(\partial t/\partial x)$ and $v(\partial t/\partial y)$ in equation (1) take into account transfer of property t by the secondary flow.

Equation (1) in a dimensionless form reads

$$Pe\left(U\frac{\partial T}{\partial \xi_1} + V\frac{\partial T}{\partial \xi_2}\right) - \frac{\partial}{\partial \xi_1}\left(1 + \frac{\varepsilon_x}{a}\right)\frac{\partial T}{\partial \xi_1} - \frac{\partial}{\partial \xi_2}\left(1 + \frac{\varepsilon_y}{a}\right)\frac{\partial T}{\partial \xi_2} = 2W. \quad (2)$$

The determination of temperature distribution in fluid flow is reduced to the solution of equation (2) in the region $0 \leqslant \xi_1 \leqslant 1, 0 \leqslant \xi_2 \leqslant 1$ with the following boundary conditions for T

$$\xi_1 = 0 \quad \frac{\partial T}{\partial \xi_1} = 0 \qquad \xi_2 = 0 \quad \frac{\partial T}{\partial \xi_2} = 0$$
$$\xi_1 = 1 \quad \frac{\partial T}{\partial \xi_1} = -1 \quad \xi_2 = 1 \quad \frac{\partial T}{\partial \xi_2} = -1 \quad (3)$$
$$\xi_1 = \xi_2 = 0 \quad T = 0.$$

3. ALGORITHM OF SOLUTION

The problem is solved by the difference method. The region $0 \leqslant \xi_1 \leqslant 1$, $0 \leqslant \xi_2 \leqslant 1$ is covered by a grid of coordinate straight lines ξ_{1i}, ξ_{2k} ($i = 1, 2, \ldots, h$, $k = 1, 2, \ldots, h$). The difference analog of equation (2) is set up by a substitution of differential operators by difference operators.

$$-a_{ik}T_{i-1k} - c_{ik}T_{i+1k} - b_{ik}T_{ik-1} - d_{ik}T_{ik+1} + e_{ik}T_{ik} = f_{ik}$$

$$a_{ik} = \frac{\omega_i}{1 + 0{\cdot}5\,Peh|U_{ik}|} + \frac{Pe}{h}\cdot\frac{|U_{ik}| + U_{ik}}{2}$$

$$c_{ik} = \frac{\omega_{i+1}}{1 + 0{\cdot}5\,Peh|U_{ik}|} - \frac{Pe}{h}\cdot\frac{U_{ik} - |U_{ik}|}{2}$$

$$\omega_i = \frac{2}{h^2}\cdot\frac{[1 + (\varepsilon_x/a)]_{i-1k}\cdot[1 + (\varepsilon_x/a)]_{ik}}{[1 + (\varepsilon_x/a)]_{i-1k} + [1 + (\varepsilon_x/a)]_{ik}}$$

$$b_{ik} = \frac{\omega_k}{1 + 0{\cdot}5\,Peh|V_{ik}|} + \frac{Pe}{h}\cdot\frac{|V_{ik}| + V_{ik}}{2}$$

$$d_{ik} = \frac{\omega_{k+1}}{1 + 0{\cdot}5\,Peh|V_{ik}|} - \frac{Pe\,V_{ik} - |V_{ik}|}{h \quad\quad 2}$$

$$\omega_k = \frac{2}{h^2}\cdot\frac{[1 + (\varepsilon_y/a)]_{ik-1}\cdot[1 + (\varepsilon_y/a)]_{ik}}{[1 + (\varepsilon_y/a)]_{ik-1} + [1 + (\varepsilon_y/a)]_{ik}}$$

$$e_{ik} = a_{ik} + b_{ik} + c_{ik} + d_{ik}. \quad (4)$$

The functions $(\varepsilon_x/a)(\xi_1, \xi_2)$, $(\varepsilon_y/a)(\xi_1, \xi_2)$ are calculated by the procedure, proposed in [8], allowing for the anisotropy of heat transfer in turbulent flow. For the calculation of the velocity component $W(\xi_1, \xi_2, Re)$ and also of those of the secondary flow U and V use is made of the algorithm from [6]. Realization of the conditions (3) for T at the boundary of the region of calculation is reduced to recalculation of the coefficients a_{ik}, b_{ik}, c_{ik}, d_{ik} and of the right-hand side f_{ik} of equation (4) at the points i, k, adjoining the boundary.

The scheme for the solution of equation (4), used in the present paper, is

$$(\sigma + e_i)\,T_{ik}^{l+\frac{1}{2}} - a_{ik}T_{i-1k}^{l+\frac{1}{2}} - c_{ik}T_{i+1k}^{l+\frac{1}{2}} = (\sigma - e_k)\,T_{ik}^{e} + b_{ik}T_{ik-1}^{e} + d_{ik}T_{ik+1}^{e} + f_{ik}$$

$$(\sigma + e_k)\,T_{ik}^{l+\frac{1}{2}} - b_{ik}T_{ik-1}^{e+\frac{1}{2}} - d_{ik}T_{ik+1}^{e+\frac{1}{2}} = \sigma T_{ik}^{e} - b_{ik}T_{ik-1}^{e} - d_{ik}T_{ik+1}^{e}$$

$$e_i \geqslant a_{ik} + c_{ik}, \quad e_k \geqslant b_{ik} + d_{ik}$$

$$e_{ik}T_{ik}^{e+\frac{3}{4}} - b_{ik}T_{ik-1}^{e+\frac{3}{4}} - d_{ik}T_{ik+1}^{e+\frac{3}{4}}$$

$$= f_{ik} + (DT^{e+\frac{1}{2}})_{ik}$$

$$T_{ik}^{e+1} = \alpha T_{i-1k}^{e+1} + \xi_{ik}\, T_{i+1k}^{e+1} + T_{ik}^{e+\frac{3}{4}}$$

$$\alpha_{ik} = \frac{a_{ik}}{e_{ik}}, \quad \xi_{ik} = \frac{c_{ik}}{e_{ik}}$$

$$(DT)_{ik} = b_{ik}(\alpha_{ik-1}\, T_{i-1k-1} + \xi_{ik-1}\, T_{i+1k-1}) + + d_{ik}(\alpha_{ik+1}\, T_{i-1k+1} + \xi_{ik+1}\, T_{i+1k+1}). \quad (5)$$

The optimum value of the relaxation parameter σ has been selected experimentally, by carrying out the trial calculation. Algorithm (5) has turned to be rather effective. Thus at the assembly of 576 points for optimum σ the inequality

$$\max_{ik} \left| \frac{T_{ik}^{e+1} - T_{ik}^{e}}{T_{ik}^{e}} \right| < 0{\cdot}001$$

is fulfilled after 25 iterations (one iteration is considered to be a transition from the l-iterative level to the $(l + 1)$ level.

4. CALCULATION RESULTS

Calculation results of the temperature field in a fluid flow ($Re = 30000$) are presented in Figs. 1–4. The calculations are carried out for Pr of 4·35 (water) and 0·027 (mercury). The results of the measurements of the temperature field in a water flow for the case under consideration are given in Fig. 5 [7]. Figure 6 reports the results of calculation of hydrodynamic characteristics of the secondary flow.

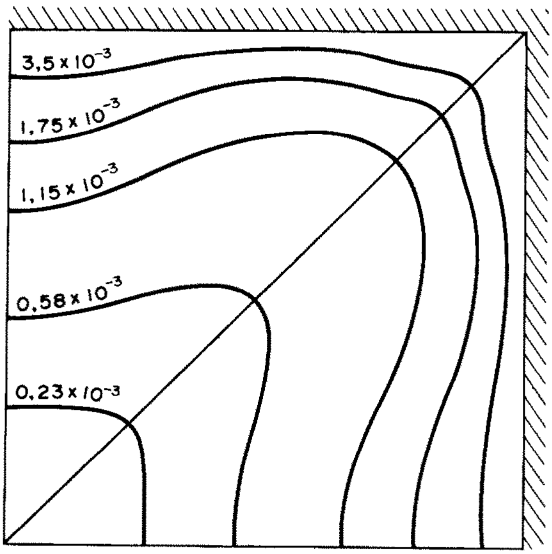

FIG. 1.

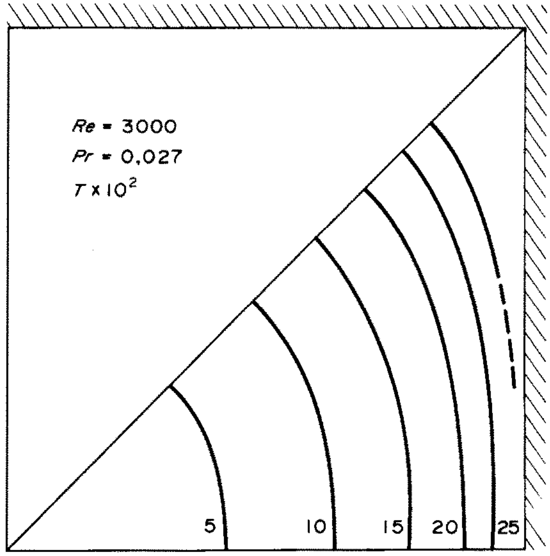

FIG. 2.

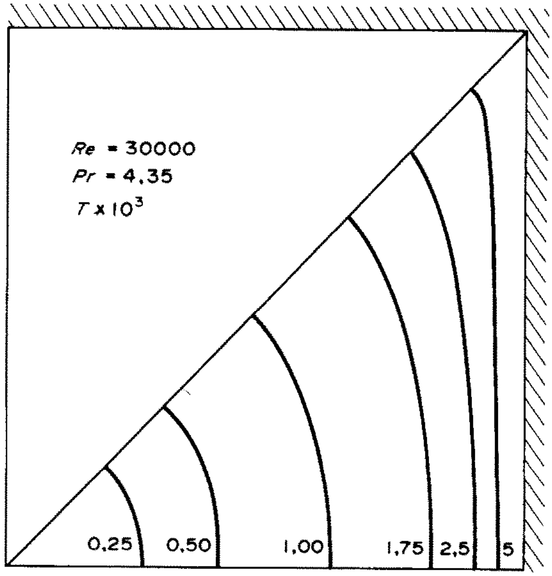

FIG. 3.

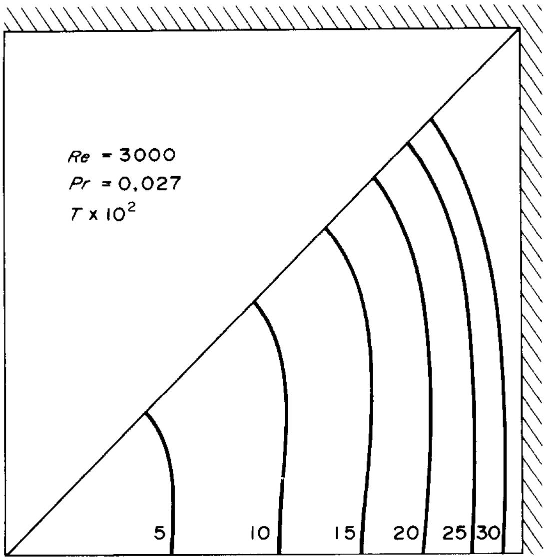

FIG. 4.

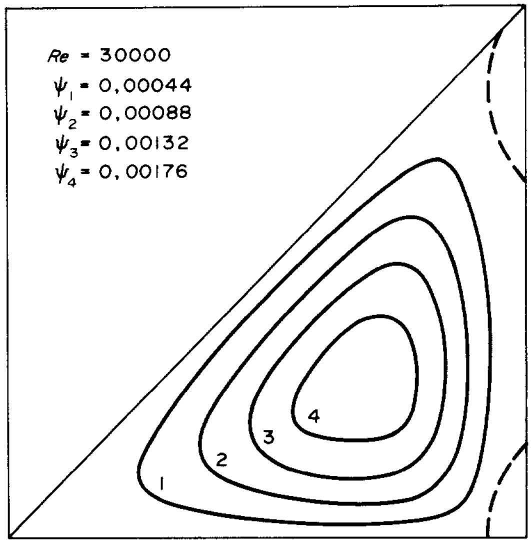

FIG. 6. Stream lines of a secondary flow in a duct cross-section Re = 30000.

The temperature field in a water flow, calculated without allowance for the secondary flow, i.e. at $u = v = 0$ in equation (1) is represented in Fig. 3. Attention is drawn to the fact that the calculation of the temperature field with inclusion of the secondary flow (Fig. 1) yields results which are in closer aggreement with measured data (Fig. 5) both with respect to the type of isotherms and to the absolute values of temperature. For small values of Pr (Figs. 2 and 4) the allowance for secondary flow is not important.

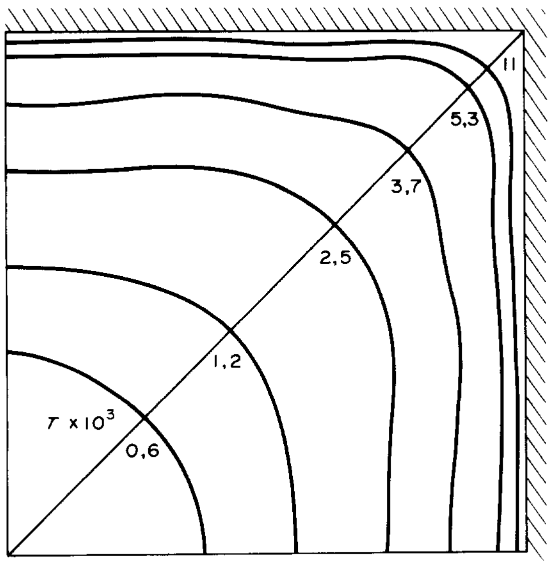

FIG. 5. Temperature field in flow of water, experiment [7], Pr = 4.35.

Comparison of the calculated results of temperature fields in a square duct with experimental measurements shows that in solving the problem the basic factors, connected with the effect of the secondary flow on the processes involved in the transfer of a scalar property in turbulent fluid flow, have been taken into consideration.

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Abstract—Calculation method is proposed for the determination of the temperature field in a fluid in noncircular ducts. The calculation results are given for the case of turbulent flow in a square duct. For comparison results of measurements are presented of a temperature field in fluid flow under conditions close to those assumed in the calculations.

CALCUL DU TRANSFERT THERMIQUE DANS UN ÉCOULEMENT TURBULENT AVEC MOUVEMENT SECONDAIRE

Résumé—On présente dans cet article, une méthode de calcul du champ de température pour un fluide à l'intérieur de conduits non circulaires. Les résultats du calcul sont donnés dans le cas d'un écoulement à l'intérieur d'un conduit carré. Afin d'établir une comparison, on présente les résultats de mesures d'un champ de température dans un écoulement de fluide pour des conditions proches de celles du calcul.

BERECHNUNG DES WÄRMEÜBERGANGS IN TURBULENTER STRÖMUNG BEI BERÜCKSICHTIGUNG VON SEKUNDÄRSTRÖMUNGEN

Zusammenfassung—Es wird ein Berechnungsverfahren vorgeschlagen für das Temperaturfeld bei Strömungen durch Kanäle mit nichtkreisförmigem Querschnitt. Rechenergebnisse sind angegeben für eine turbulente Strömung in einem quadratischen Querschnitt. Zum Vergleich werden Messergebnisse für das Temperaturfeld in einer Strömung angegeben, deren Verhältnisse ähnlich denen der berechneten sind.